Grade 8 Solving Linear Equations

8.PR.2	
Model and solve problems using linear equations of the form: • $ax = b$ • $x/a = b$, a not equal 0 • $ax + b = c$ • $x/a + b = c$, a not equal 0 • $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c are integers.	1. Model a problem with a linear equation and solve the equation using concrete models. 2. Verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams, and substitution. 3. Draw a visual representation of the steps used to solve a linear equation and record each step symbolically. 4. Solve a linear equation symbolically. 5. Identify and correct an error in an incorrect solution of a linear equation. 6. Apply the distributive property to solve a linear equation (e.g., $2[x + 3] = 5$; $2x + 6 = 5$; …) 7. Solve a problem using a linear equation and record the process.

Clarification of the outcome:

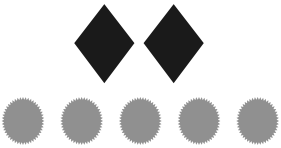
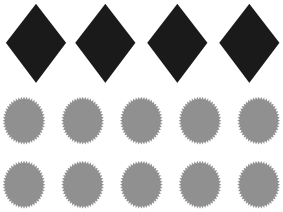
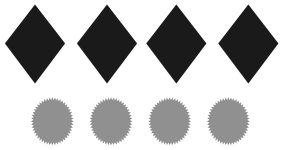
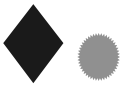
- ✦ This outcome concerns understanding and being able to solve linear equations that require one or more steps to solve. This outcome is partly a continuation of the grade 7 outcome equation types $ax = b$, $x/a = b$, and $ax + b = c$ (but now for integers). There are two new equation types in grade 8: $x/a + b = c$ and $a(x + b) = c$ (for integers).

Required close-to-at-hand prior knowledge:

- ✤ Can solve, or mostly can solve the grade 7 equation types: $ax = b$, $x/a = b$, and $ax + b = c$, for whole numbers.
- ✤ Understands brackets as containers.
- ✤ Understands the distributive principle.
- ✤ Understands that add and subtract are opposite (inverse) operations and that multiply and divide are opposite (inverse) operations.

SET SCENE stage

Begin by showing students a suitable number trick (see below). Explain what each column is about. The numerical example column concerns using s specific number. The pictorial column concerns generalizing the results of the steps using somewhat crude notation (pictures that cannot be readily worked with). The algebra column concerns generalizing the results of the steps using a notation system that is well suited to working with.

STEP	NUMERICAL EXAMPLE	PICTORIAL REPRESENTATION	ALGEBRAIC FORM
Choose a number	5		x
Double	10		2x
Add 5	15		2x + 5
Double	30		4x +10
Subtract 6	24		4x + 4
Divide by 4	6		x + 1
Subtract number first chosen	1		1

The problem task to present to students:

Provide students with a number trick sheet to complete that is comparable to the one that was demonstrated (see below). Have students use different numerical examples.

STEP	NUMERICAL EXAMPLE	PICTORIAL REPRESENTATION	ALGEBRAIC FORM
Choose a number			
Add 1			
Times 3			
Add 6			
Divide by 3			
Subtract 3			

Comments:

The purpose of the task is present a situation that hopefully provides a partial reason for learning algebra and that provides equation-solving situations.

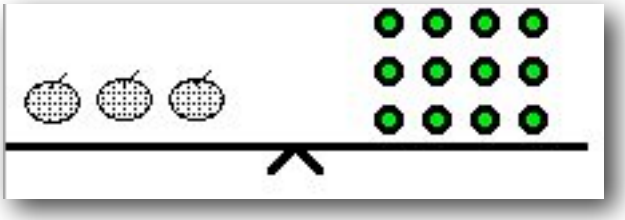
DEVELOP stage

Activity 1: Revisits SET SCENE

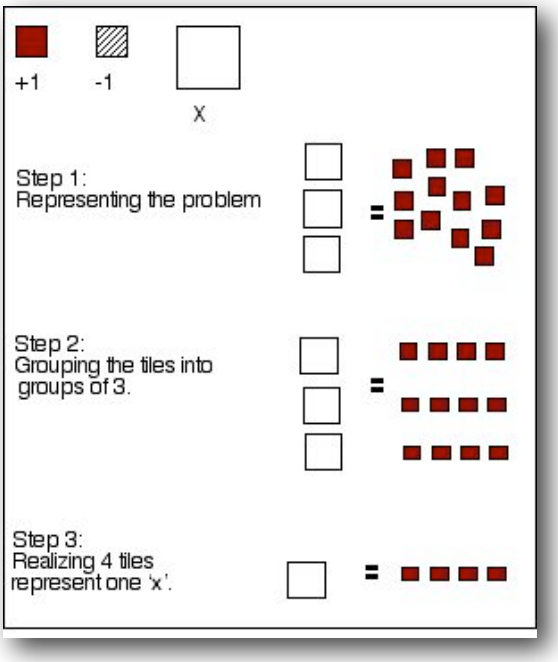
Ask selected students to present their completed number trick sheet. Ensure that the three columns are correctly completed. Discuss why the pictorial and algebraic representations generalize the situation to "prove" the result for the number trick while a numerical example does not "prove". Mention that we will come back to the algebra form later on in the lesson.

Activity 2: Addresses achievement indicators 1, 2, 3, 4, and 7. (partly revisits grade 7)

- Ask students to represent the problem, *Three apples balance 12 grapes. How many grapes will balance one apple?*, as an equation. [Expect: 3x = 12]
- Have them solve the equation by representing it in balance beam form (see diagram for the start of the solution). Discuss their solutions.



- Have them solve the equation by using algebra tiles (see diagram). Discuss their solutions.



- Discuss an algebraic approach to solving the equation. Ensure they understand the linkage between the algebraic approach and the balance beam and algebra tiles approaches. Have students check the solution by substituting the numerical value for the variable ('x', in this example).

$$4x = 12$$

$$4x/4 = 12/4$$

$$x = 3$$

- Provide two other equations of the form: ax = b (whole numbers only). Ask students to solve them algebraically. Ensure they realize that division by 'a' involves doing the opposite operation to move 'a' away from the variable 'x'. Have them verify the solution by substitution.
- Provide three other equations of the form: ax = b (involve integers: e.g. -3x = 15). Ask students to solve them algebraically. Ensure they realize that division by 'a' involves doing the opposite operation to move 'a' away from the variable 'x'.

Activity 3: Addresses achievement indicators 4 and 7. (partly revisits grade 7)

- Ask students to represent the problem, *A half a glass of water holds 35 ml of water. How much water does a full glass hold?*, as an equation.
- Discuss an algebraic approach to solving the equation. Ask students to solve the equation algebraically. Ensure they realize that multiplication by '2' involves doing the opposite operation to move '2' away from the variable 'x'. Have them verify the solution by substitution.
- Provide two other equations of the form: x/a = b. Ask students to solve them algebraically. Ensure they realize that multiplication by 'a' involves doing the opposite operation to move 'a' away from the variable 'x'. Have them verify the solution by substitution.
- Provide three other equations of the form: x/a = b (involve integers: e.g. x/-4 = -6). Ask students to solve them algebraically. Ensure they realize that multiplication by 'a' involves doing the opposite operation to move 'a' away from the variable 'x'.

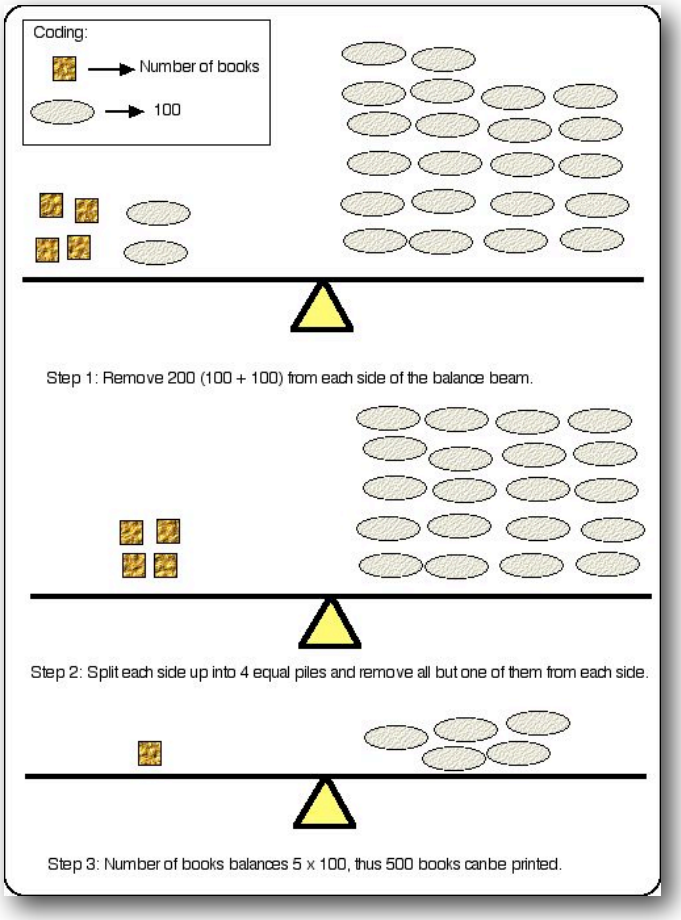
Activity 4: Addresses achievement indicators 1, 2, 3, 4, and 7. (partly revisits grade 7)

- Pose the problem:

  The school year book costs money to produce. Each book costs $4 to print. As well there is a set up cost of $200 that must be paid no matter how many year books are printed. The year book committee has $2200 to spend. How many year books can be printed?

- Ask students to express the problem as an equation. [Expect: 4N + 200 = 2200.]

- Have students solve the equation by using the algebra tiles or the balance beam model. They will need to code the model in an appropriate way. Refer to the diagram for a balance beam approach. Discuss their solution methods.



- Provide two other equations of the form: ax + b = c. Ask students to solve them algebraically. Ensure they realize that division by 'a' and subtraction of 'b' involves doing the opposite operation to move 'a' and 'b' away from the variable 'x'. Have them verify the solutions by substitution.

- Provide four other equations of the form: ax + b = c (involve integers: e.g. -2x - 3 = 7). Ask students to solve them algebraically. Ensure they realize that division by 'a' and subtraction of 'b' involves doing the opposite operation to move 'a' and 'b' away from the variable 'x'. Have them verify the solutions by substitution.

Activity 5: Addresses achievement indicators 2, 4, and 7.

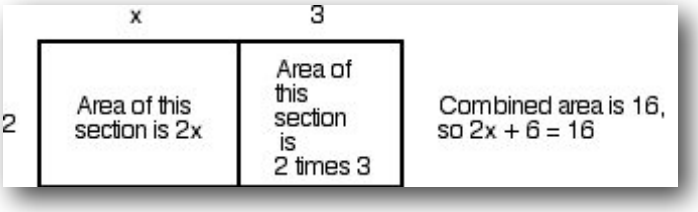
- Present students with the equation type, x/a + b = c, where a, b, and c are all positive (e.g. x/3 + 4 = 9). Discuss using the opposite operation to remove 3 and 4. Have students solve the equation algebraically. Have them verify the solution by substitution.
- Present students with at least three equations where a, b and c are all positive. Have students solve the equation algebraically. Discuss their solution methods. Ensure they are thinking in terms of the opposite operation. Have them verify the solutions by substitution.
- Present students with at four equations where at least one of a, b and c is negative (e.g. x/-2 + 4 = -1). Have students solve the equation algebraically. Discuss their solution methods. Ensure they are thinking in terms of the opposite operation. Have them verify the solutions by substitution.

Activity 6: Addresses achievement indicators 4, 5, and 7.

- Organize students into groups. Provide each group with at least two equations of the type, ax + b = c, and of the type, x/a + b = c (where a, b, and c are integers) and the algebraic solutions for solving the equations. Each solution has at least one error in it. Ask the groups to identify the errors and to correct the solutions. Ask each group to present the identified error(s) and the correct solution for one of the equations.

Activity 7: Addresses achievement indicators 2, 4, 6, and 7.

- Present students with the equation type, a(x + b) = c, where a, b, and c are all positive (e.g. 2(x + 3) = 16). Discuss using the distributive principle for dealing with the multiplier in front of the bracket. Use the rectangle model (see diagram) to help students realize that the multiplier (e.g. 2) must be multiplied times BOTH parts inside the bracket (e.g. x and 3). [Refer to: Gr 4 Multiplication algorithm (4.N.6)] Have students solve the equation algebraically. Ensure they are thinking in terms of OPPOSITE operation. Have them verify the solutions by substitution. Repeat for two other equations of the type, a(x + b) = c, where a, b, and c are all positive.



- Present students with the equation type, a(x + b) = c, where at least one of a, b, and c are negative (e.g. -2(x + 3) = -4). Discuss using the distributive principle for dealing with the multiplier in front of the bracket. Have students solve the equation algebraically. Ensure they are thinking in terms of OPPOSITE operation. Have them verify the solution by substitution. Repeat for two other equations of the type a(x + b) = c where at least one of a, b, and c is negative.

Activity 8: Addresses achievement indicators 1, 2, 3, 4, and revisits SET SCENE.

- Revisit the SET SCENE number trick. For each step, have students verify that the value of x is the starting number. Here is an example of what this means. Suppose the a student used '2' as the starting number. Suppose we consider step 4 (add 6). The arithmetic for this step works out to be 15 (when the starting number is 2). The algebra entry at step 4 is 3x + 9. The equation then for step 4 would be: 3x + 9 = 15. If you solve it for x, you get 2, the starting number.

Activity 9: Addresses achievement indicators 2, 4, 5, and 6.

- Organize students into groups. Provide each group with algebraic solutions to equations for each of the types included in the achievement indicators. Each solution has at least one error in it. Ask the groups to identify the errors and then to correct the solutions. Ask each group to present the identified error(s) for a solution to one of the equations and the corrected solution for it, and to verify the solution by substitution.

Activity 10: Addresses achievement indicators 2, 4, 6, and 7.

- Organize students into groups. Have each group make up one word problem for each equation of the form ax + b = c, x/a + b = c, and a(x + b) = c. Have each group solve its own word problems. Have selected groups present their word problems, equations, and solutions.

Activity 11: Assessment of teaching.

- Provide students with <u>one</u> equation for <u>each form</u> ('ax + b = c', x/a + b = c, and a(x + b) = c) where a, b, and c are integers. Have them solve the equations algebraically.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

Use x = 5 for x in each equation. Decide whether x needs to be greater than or less than 5 to make the equation true.

a) $x + 7 = 10$

b) $x - 3 = 12$

c) $3x = 50$

d) $x/3 = 4$

e) $2x + 7 = 8$

f) $x/2 + 3 = 10$

g) $3(x + 1) = 10$

Question 2.

Circle the equations that are another way of writing $8x + 3 = 51$.

a) $3 + 8x = 51$

b) $51 = 8x + 3$

c) $51 - 3 = 8x$

d) $8x = 51 + 3$

e) $x = (51 - 3)/8$

f) $8x + 51 = 3$

Question 3.

The same value of x solves all the equations. What are the missing numbers?

$3x = 15$, $3x + \_\_ = 16$, $3x + 4 = \_\_$, $3x - \_\_ = 11$, $\_\_ x = 150$, $x/\_\_ = 5$

Question 4.

Solve each equation.

a) $x - 3 = 10$

b) $x + 7 = -1$

c) $3x = -21$

d) $-x/2 = 14$

e) $2x - 1 = 5$

f) $x/3 + 4 = -1$

g) $4(x - 3) = -16$

MAINTAIN stage

Mini-task example

Every so often:

- Provide students with two equations. Have them solve them and present solutions.

Rich-task example

Have students solve the following problem by writing an appropriate equation and then solving it.

Bill is a contestant in a pie eating contest. He has a huge pie in front of him that he has to eat. Bill begins by eating a certain percent of the pie. He takes a breather and then eats the percent of the pie that he first ate along with another 20% of the original pie. He takes another breather. Finally, he eats the percent of the pie that he first ate along with another 5% of the original pie. When Bill finished, he had eaten the whole pie. What percent of the pie did Bill eat to begin with?

Comments

This is a rich-task because it integrates problem solving, equation solving, and percent.